Relation-aware Exploration of Fiber Bundles

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Abstract—This paper

Index Terms—DTI, fiber bundle, uncertainty, spatial relation.

Introduction

1 Related Work

2 Our approach

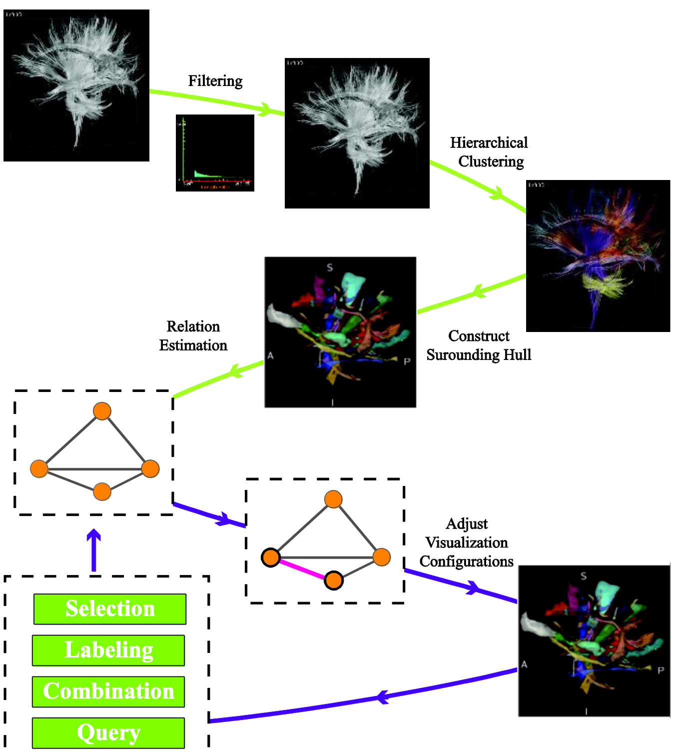

Figure 1 The pipeline of our approach.

Our proposed relation-aware fiber exploration approach consists of several process. Figure 1 shows a schematic overview of our approach. Roughly speaking, this pipeline has two stages. The pre-processing stage consists of processes along the green line. The interactive exploration loop is highlighted in purple.

Given a fiber model, our approach first remove all fibers of non-interest, e.g the short fibers. Then, an agglomerative hierarchical clustering method is employed to group fibers into different fiber bundles. We also build a set of geometrical hulls to approximate the shape and orientation of each fiber bundle. Based on the relation logic and measurements, relations between geometrical hulls are derived, which also describes the relations of fiber bundles. At last, the relations and the fiber bundles are represented by a node-link diagram, called *relation graph*.

In the exploration loop, interactions for the 3D fibers are performed in the relation graph. Users can easily select a fiber bundle, add labels for a fiber bundle, and combine different fiber bundles. More details about these processes will be discussed in the following sections.

2.1 Hierarchical Clustering

The core of clustering is to estimate the distance between fibers. However, the conventional metrics [cite] only considers the positions of fibers. All other features including shape, orientation, and scale are neglected. Besides that, these metrics are independent of re-parameterizations of fibers. In this paper, we choose to estimate the distance between fibers in a full-feature space (position, shape, orientation, and scale) associated with a Riemanian metric.

Let $\beta\colon [0,1] \to \mathbb{R}^3$ be a parameterized curve, representing a fiber. The set of all re-parameterization for a fiber is $\Gamma = \{\gamma\colon [0,1] \to [0,1] | \gamma(0) = 0, \gamma(1) = 1\}$. For any $\gamma \in \Gamma$, $(\beta, \gamma)(\tau) = \beta(\gamma(\tau))$ is a re-parameterization of $\beta(\tau)$. To enable comparison of fibers in a full-feature space, $\beta(\tau)$ is represented by a square-root function (**SRF**):

$$h(\tau) = \sqrt{\|\dot{\beta}(\tau)\|}\beta(\tau), h\colon [0,1] \to \mathbb{R}^3 ,$$

where $\dot{\beta}(\cdot)$ is the velocity function. Correspondingly, the SRF for a re-parameterized fiber $(\beta, \gamma)(\tau)$ changes to $(h, \gamma)(\tau) \stackrel{\text{def}}{=} h(\gamma(\tau))\sqrt{\dot{\gamma}(\tau)}$.

Due to re-parameterization, $\|\beta_1 - \beta_2\| \neq \|(\beta_1, \gamma) - (\beta_2, \gamma)\|$ in general. By representing fibers with SRF, $\|(h_1, \gamma) - (h_2, \gamma)\| = \|h_1 - h_2\|$. The proof of this conclusion can be found in [cite]. Because of this equality, the distance between a pair of fibers is defined as:

$$d(\beta_1, \beta_2) = \min_{\gamma \in \Gamma}(\|h_1, (h_2, \gamma)\|)$$

The standard dynamic programming can be employed to solve this minimization. The result quantifies the similarities of fiber in terms of feature mentioned before.

With this new fiber distance measure, the fibers in the given fiber model are clustered into bundles with the single-linkage algorithm [cite]. Compared with some optimization-base clustering methods, single-linkage algorithm ensures a minimum distance threshold between clusters and the hierarchical structure of clusters naturally assists the exploration of clustering results at different levels-of-abstract [cite]. Each cluster of fibers is also called a fiber bundle.

2.2 Surrounding Hull

For each fiber bundle, we discretize all fibers into vertices and build a geometric surrounding hull by means of alpha shape algorithm [cite].

2.3 Relation Estimation

Typically, different anatomical structures do not function independently. They coordinate with each other. Spatial relations have significant implications on how they function together.

2.3.1 Relation Measures

Region Connection Calculus [cite] is a widely used region-based approach in spatial reasoning. It derives a set of spatial relations by

using algebraic logic based on the connectivity property. Give any to object, they can be first classified as Connected (**C**) or Disconnected (**DC**). Connected means that the topological closures of them share a common point. Then, they can be further classified into more specific relations, e.g. Part of (**P**), Proper Part of (**PP**), Overlapping (**O**), and so on (see Table 1).

Table 1 Different types of relation based on RCC.

Relation	Description	logical expression
$\mathbf{DC}(s_i, s_j)$	disconnected	$\neg\mathbf{C}(s_i, s_j)$
$\mathbf{P}(s_i, s_j)$	part of	$\forall s_k[\mathbf{C}(s_k, s_i) \rightarrow \mathbf{C}(s_k, s_j)]$
$\mathbf{PP}(s_i, s_j)$	proper part of	$\mathbf{P}(s_i, s_j) \wedge \neg\mathbf{P}(s_j, s_i)$
$\mathbf{EQ}(s_i, s_j)$	identical with	$\mathbf{P}(s_i, s_j) \wedge \mathbf{P}(s_j, s_i)$
$\mathbf{O}(s_i, s_j)$	overlapping	$\exists s_k[\mathbf{P}(s_k, s_i) \wedge \mathbf{P}(s_k, s_j)]$
$\mathbf{DR}(s_i, s_j)$	discrete from	$\neg\mathbf{O}(s_i, s_j)$
$\mathbf{PO}(s_i, s_j)$	partially overlapping	$\mathbf{O}(s_i, s_j) \wedge \neg\mathbf{P}(s_i, s_j) \wedge \neg\mathbf{P}(s_j, s_i)$
$\mathbf{EC}(s_i, s_j)$	externally connected	$\mathbf{C}(s_i, s_j) \wedge \neg\mathbf{O}(s_i, s_j)$
$\mathbf{TPP}(s_i, s_j)$	tangential proper part	$\mathbf{PP}(s_i, s_j) \wedge \exists s_k[\mathbf{EC}(s_k, s_i) \wedge \mathbf{EC}(s_k, s_j)]$
$\mathbf{NTPP}(s_i, s_j)$	nontangential proper part	$\mathbf{PP}(s_i, s_j) \wedge \neg\exists s_k[\mathbf{EC}(s_k, s_i) \wedge \mathbf{EC}(s_k, s_j)]$

For simplicity, we characterize the relations between fiber bundles into four categories: DC, P, PO, and EC. If two fiber bundles are far from each other, their relation might be DC. One limitations of RCC is that it can only characterize the shape relation between fiber bundles. The orientation of fiber bundles is neglected.

To The orientation relation is defined based on the line segments passing through the average starting points to the average ending points for each bundle (See Figure 2 as an example). In general, three types of orientation relation is defined: Parallel, Intersect, and Extent.

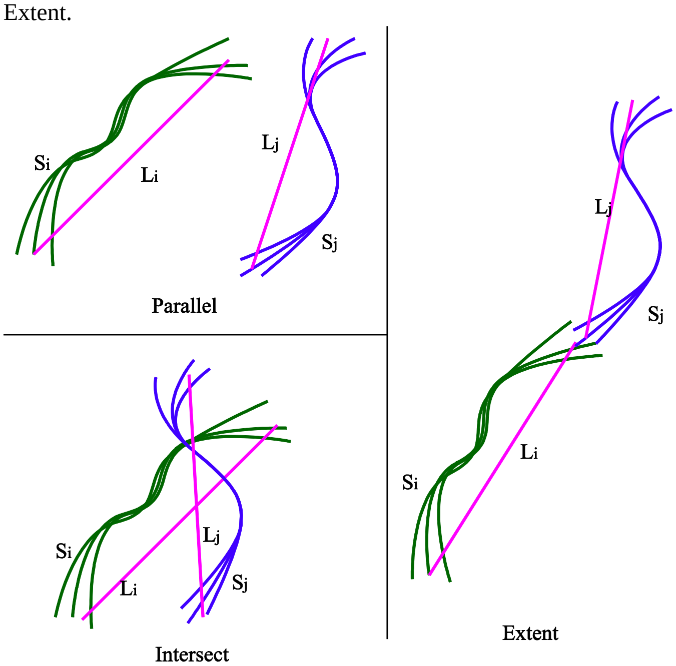

Figure 2 Different types of orientation relation between fiber bundle S_i and S_j which is estimated by line segments L_i and L_j.

2.3.2 Relation Graph

Given the set of relations measured in Section 2.3.1, we need to intuitively present it to users for exploration. Similar to [cite], we also employ an enhanced node-link diagram to facilitate visualization and exploration of fiber bundles and their relations. More specifically, the nodes represent fiber bundles. The edges indicate the relations between fiber bundles. In order to encode both shape relation and orientation relation on a single edge, we chose to use color for shape relation and glyph for orientation relation (see Figure 3 as an example).

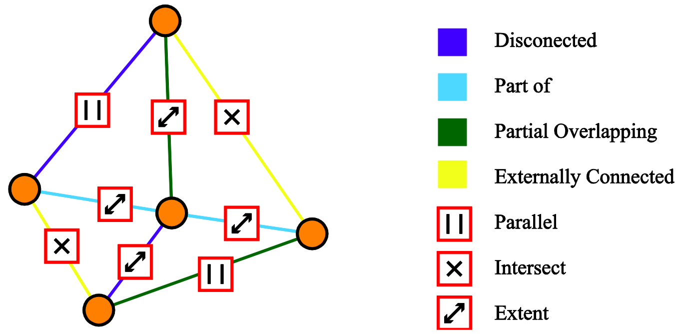

Figure 3 An example of relation graph.

3 Results and Discussion

4 Expert Evaluation

References

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